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# Regulatory Risk under Optimal Incentive Regulation

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# Regulatory Risk under Optimal Incentive Regulation

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## Abstract

The paper provides a tractable, analytical framework to study regulatory risk under optimal incentive regulation. Regulatory risk is captured by uncertainty about the policy variables in the regulator's objective function: weights attached to profits and costs of public funds. Results are as follows: 1) The regulator's reaction to regulatory risk depends on the curvature of the aggregate demand function. 2) It yields a positive information rent effect exactly when demand is convex. 3) Firms benefit from regulatory risk exactly when demand is convex. 4) Consumers' risk preferences tend to contradict the firm's. 5) Benevolent regulators always prefer regulatory risk and these preferences may contradict both the firm's and consumers' preferences.

*Keywords: optimal incentive regulation, regulatory risk, procurement, information rents*

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# 1 Introduction

Regulatory risk reflects the uncertainty behind new or changing regulation over time. As illustrated in Table I, recent surveys reveal that firms view regulatory risk as one the greatest threats to their business.<sup>1</sup> Emphasizing the importance of regulatory risk, the Ernst&Young 2008 survey on strategic business risk proclaimed regulatory and compliance risk as “the greatest strategic challenge facing leading global businesses in 2008”. In popular debate, regulatory risk is seen as an important impediment to a firm’s long term investment and, therefore, to hinder economic growth. Regulators acknowledge that regulatory risk may impede effective regulation but downplay its influence on their actual policies.<sup>2</sup>

Despite its judged importance by practitioners, regulatory risk has received little attention in economic theory. In particular, its economic effects have not been studied in modern theories of optimal regulation. Without such studies, our understanding of the problem remains limited. This paper contributes to filling this gap between practise and theory by providing a tractable, analytical framework to study regulatory risk. The framework is based on the seminal regulation models of Baron and Myerson (1982) and Laffont and Tirole (1986). These models include two natural regulatory variables: the relative weight of the firm’s profits in the regulator’s objective function and the cost of public funds.<sup>3,4</sup> I show that introducing uncertainty concerning these regulatory variables yields a formal model of regulatory risk and enables an evaluation of the attitudes of different economic agents towards regulatory risk.

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<sup>1</sup>Surveys freely available at <http://graphics.eiu.com> and <http://www.ey.com>. See also DLA Piper “European Regulatory Risk Awareness Survey 2007” accessible at <http://www.dlapiper.com>.

<sup>2</sup>E.g., “What Regulatory Risk” by Paul Plummer, ORR Chief Economist: <http://www.rail-reg.gov.uk/upload/pdf/utiljrnl.pdf> (11-15-2008)

<sup>3</sup>Baron and Myerson (1982) introduced the regulation models that are based on the relative weight of the firm’s profits in the regulator’s objective function. Laffont and Tirole (1986) developed regulation models that are based on a strictly positive cost of public funds. I follow the approach of Armstrong and Sappington (2008, p.1563) who offer an integration of these two models.

<sup>4</sup>Clearly, regulatory risk may also be modeled differently. The paper’s conclusion discusses and points to such alternative modeling strategies.

Table I

*Top 5 risks to business according to EUI and Ernst&Young reports on business risks.*

Rank	EUI 2005	E&J 2008	E&J 2009
1	Regulatory risk	Regulatory and compliance Risk	Credit crunch
2	IT network risk	Global and financial shocks	Regulatory and compliance
3	Human capital risk	Aging consumers and workforce	Deepening recession
4	Reputational risk	Emerging markets	Radical greening
5	Market risk	Industry consolidation	Non-traditional entrants

The framework reveals that, contrary to the popular view, regulatory risk may actually benefit firms. This occurs exactly when aggregate consumer demand is convex. The curvature of the demand function plays a crucial role, because it determines the direction of an *information rent effect* of regulatory risk. In particular, I show that, in canonical models of optimal incentive regulation, the regulator reacts to an increase in regulatory risk by increasing output when demand is convex. This reaction implies higher information rents and benefits the firm. In contrast, the consumer, who is ultimately paying for these rents, is hurt by a positive information rent effect. For concave demand, the results are reversed. The regulator reacts to regulatory risk by reducing output and, therefore, the information rent effect is negative. This hurts the firm, but benefits consumers.

I show that the information rent effect uniquely determines the firm's risk attitude towards regulatory risk. Hence, with convex demand the firm likes regulatory risk and, for concave demand, the firm dislikes it. In contrast, the consumer's attitude towards regulatory risk is more complex. Due to decreasing marginal utility, the consumer has a natural aversion towards fluctuations in output. Hence, in addition to the information rent effect, the consumer's attitude towards regulatory risk also depends on how it affects output fluctuations. Notwithstanding this additional effect, the analysis reveals that with convex demand consumers are hurt by uncertainty about the weight which the regulator attaches to profits, while consumers like risk about the cost of public funds when demand is concave. The con-

sumers' overall attitude towards regulatory risk depends, therefore, on the policy dimension in which the regulatory risk dominates. In general, the consumers' risk attitudes can be aligned or misaligned with those of the firm.

I further obtain that the regulator himself always benefits from regulatory risk. The intuition behind this result is similar to the classical observation of Waugh (1944), who showed that consumers may benefit from fluctuating prices. With respect to the regulator, this insight leads to the paradoxical result that the regulator's risk preferences may contradict both those of the firm and consumers, even though he is "benevolent" in the sense that his objective function is a weighted sum of the consumers' surplus and the firms profits. The result obtains when there is uncertainty about the weight with which the regulator evaluates profits, because the uncertainty about this weight destroys the natural alignment between the preferences of the regulator and those of the consumers and firm. The result implies that a benevolent regulator may not want to lower the degree of regulatory risk even if this is in the interest of both consumers and the regulated firm. This has the policy implication that additional incentives are needed, such as an explicit directive to minimize regulatory risk, to induce the regulator to reduce regulatory risk.

How reasonable is it to model regulatory risk as uncertainty about the cost of public funds and the regulator's weight attached to profits? First of all, these policy variables have no direct economic effect on firms or consumers and only matter in relation to the regulation problem itself. Hence, from a theoretical perspective, uncertainty about these variables is the purest form of regulatory risk one may consider. From a more practical view, the relevance of the model depends, of course, on whether these variables are actually uncertain in practise. Auriol and Warlters (2006) show empirically that the marginal cost of public funds fluctuates much between countries. They confirm the intuition that it depends on a country's taxation system and, in particular, tax rates. In most, if not all, advanced economies tax rates change considerably and rather unpredictability in the period of just a few decades. Hence, in particular for long term investments such as power plants and infrastructure, uncertainties about the future cost of public funds are considerable. Similarly, one may argue that also the regulator's weight on profits is uncertain. Regulation is often influenced by political

considerations and political parties may have differing views about the correct weight of company profits in the regulator’s objective function. Indeed, one of the main differences between political parties in modern economies is their degree of business friendliness and their attitudes towards profits. Hence, if we acknowledge that election outcomes are uncertain, then this policy variable is uncertain for regulatory projects that outlast the typical electoral cycle.

The rest of the paper is organized as follows. The next section discusses the related literature. Section 3 introduces a regulation model based on Armstrong and Sappington (2008) that comprises the two seminal regulation models of Baron and Myerson (1982) and Laffont and Tirole (1986). Section 4 computes the optimal regulation schedule for deterministic policy variables and studies its comparative statics with respect to the policy variables. In Section 5 I then investigate the attitudes towards regulatory risk by the different economic agents: the firm, the regulator, and consumers. The paper closes with a conclusion where I discuss the different policy implications of my results. Formal proofs of the propositions are collected in the appendix.

## 2 Related Literature

Although the literature on regulation is vast, the strand of the literature that explicitly addresses the riskiness of regulation is small. Chang and Thompson (1989) analyze regulatory risk for the case of rate of return regulation. They illustrate with numerical examples that firms may benefit or lose from random errors in setting the allowed rate of return. Panteghini and Scarpa (2003) study the effect of regulatory risk on investment by comparing price-caps to profit-sharing rules. Both these papers compare ad-hoc regulation schemes rather than studying regulatory risk in an optimal regulation framework.

Regulatory risk is related to the literature on “regulatory opportunism”, e.g., Laffont and Tirole (1988, 1990), Lyon (1991), Salant and Woroch (1992), Gilbert and Newbery (1994), Lyon and Li (2003), and Lyon and Mayo (2005). This literature studies regulators

that, due to incomplete regulatory contracts, behave opportunistically. Regulatory risk and regulatory opportunism are, however, two distinct concepts. Regulatory risk involves, necessarily, uncertainty about regulatory changes, whereas regulatory opportunism does not. For instance, in Laffont and Tirole (1988; 1990) or Salant and Woroch (1992) the regulator behaves opportunistically, but the firm fully anticipates the implied future changes in regulation. Consequently, there is opportunism but no risk. The problem of regulatory opportunism may, however, lead to regulatory risk. In particular, it arises when firms are uncertain about the degree of opportunistic behavior by regulators as analyzed in Lyon and Li (2003). Regulatory risk may also arise due to other sources of uncertainty. For instance, Lyon (1991) and Gilbert and Newbery (1994) examine how rate of return regulation affects the firm's behavior when investments are risky. The risky investment leads to regulatory risk, because the regulatory outcome depends on the risky outcome of the project. Lyon and Mayo (2005) present an empirical appraisal of whether regulators in practise behave opportunistically.

The issue of regulatory risk has also been raised, albeit somewhat more informally, in the discussion of "stranded-costs". In particular, Baumol, Joskow, and Kahn (1994) and Baumol and Sidak (1995) discuss how, following an unexpected deregulation in US electricity markets, monopolistic utilities were unable to recoup their long term investments due to intensified competition. This discussion, however, takes a more ex post perspective. It mainly addresses the question whether regulated firms should be compensated *after* their market has been opened to competition. Nevertheless, some of the arguments do take a more ex ante perspective. In particular, Baumol and Sidak (1995) claim that a failure to compensate firms for their stranded-costs discourages future investment. Kolbe, Tye, and Myers (1993), Kolbe and Tye (1995, 1996), Pedell (2006) discuss whether and how stranded-costs affect the cost of capital.

An older literature, e.g., Oi (1961), Hartmann (1972), Abel (1983), Pindyck (1988), studies the role of price and cost uncertainty on firms. Although the fundamental question in my work is similar – how does risk affect firms? – one cannot extrapolate these results to a regulation framework. An important difference is that these papers study mainly competitive

markets, whereas, by its nature, the regulation framework applies more to monopolistic settings.<sup>5</sup> Also the underlying non-market mechanism behind the regulatory framework and the associated policy questions are different.

### 3 The Setup

Consider a monopolistic firm that can supply a public good in a quantity  $x$  at a constant average costs  $c \in [c_l, c_h]$ .<sup>6</sup> The exact marginal costs are private information of the firm. To all outsiders, the cost  $c$  is distributed according to the probability density function  $f(c)$  with support  $C = [c_l, c_h]$  and its associated cumulative density  $F(c)$ . I assume that the density function satisfies the monotone hazard rate property: the ratio  $h(c) \equiv F(c)/f(c)$  is non-decreasing in  $c$ . This assumption circumvents technical problems of bunching and guarantees that deterministic mechanisms are optimal.<sup>7</sup>

The regulated firm is risk neutral. This assumption is not only standard, but also eliminates any confusion that the firm's attitude towards regulatory risk is due to exogenously imposed risk preferences. In particular, the firm's profit from producing a quantity  $x$  for a lump-sum transfer  $t$  is

$$\Pi(t, x|c) = t - cx.$$

I follow the framework popularized in Laffont and Tirole (1986 and 1993) that the government buys the public good with public funds and makes it available to consumers. Public funds are raised by taxation. In particular, if the government has to raise taxes  $t$  to obtain a quantity  $x$  of the public good, the consumer surplus is

$$\Psi(t, x) = v(x) - (1 + \mu)t.$$

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<sup>5</sup>E.g., Samuelson (1972) argues that, in competitive markets, the positive partial equilibrium effects of fluctuating prices in Waugh (1944) and Oi (1961) are infeasible in general equilibrium.

<sup>6</sup>As discussed in Section 6 results do not crucially depend on the assumption of constant return to scale technologies.

<sup>7</sup>See Strausz (2006) for the latter claim.



The variable  $\mu \geq 0$  captures the social cost of public funds due to distortionary taxation. The term  $v(x)$  expresses the consumers' overall utility from the consumption of a quantity  $x$  of the public good. I assume that the consumer's marginal utility of the public good  $x$  is positive but decreasing, i.e.,  $v' > 0$  and  $v'' < 0$ . I also assume that  $v'''$  exists. Because  $v'$  represents the consumers' (inverse) aggregate demand function, the third derivative  $v'''$  has a natural interpretation: it determines the curvature of the consumers' aggregate demand function. For  $v''' > 0$ , aggregate demand is convex. For  $v''' < 0$ , it is concave.

Due to its monopolistic position, the government regulates the firm's production. Having the general public's interest at heart, the government institutionalizes a regulator with an objective function that consists of a weighted sum of consumer surplus and profits:

$$\begin{aligned} W(x, t, c | \lambda, \mu) &= \Psi(x, t) + \lambda \Pi(x, t | c) \\ &= v(x) - \lambda cx - (1 + \mu - \lambda)t. \end{aligned} \tag{1}$$

The regulator is given the task to maximize the objective function,  $W$ , under the presence of asymmetric information about the marginal cost  $c$  and the firm's outside option of zero.

The parameters  $\lambda$  and  $\mu$  are the two main policy variables that influence regulation. Hence, my regulation model follows the unifying approach of Armstrong and Sappington (2008, p.1563). It comprises the two classical sources of inefficiencies in regulation models as originally introduced by Baron and Myerson (1982) and Laffont and Tirole (1986). In Baron and Myerson (1982) the regulator attaches a weight of zero to the firm's profits and there is no distortionary taxation, i.e.,  $\lambda = \mu = 0$ . In the regulation framework of Laffont and Tirole (1986), distortions occur because the transfer is financed by taxation which leads to a deadweight loss, i.e.,  $\lambda = 1$  and  $\mu > 0$ . As motivated in the introduction, the paper's main conceptual new idea is to model regulatory risk as uncertainty about the policy variables  $\lambda$  and  $\mu$ .

## 4 Optimal Deterministic Regulation

Let us first compute the regulator's optimal regulation contract for a deterministic policy pair  $(\lambda, \mu)$ . Appealing to the revelation principle, the regulator's optimal contract is a direct mechanism,  $(t, x)$ , that gives the firm an incentive to report its true cost type  $c$ . Formally, a direct mechanism is a pair of functions  $t : [c_l, c_h] \rightarrow R$  and  $x : [c_l, c_h] \rightarrow R_+$ , where  $t(c)$  represents the transfer to the firm and  $x(c)$  the required quantity to be produced when the firm reports a cost  $c$ .

Let

$$\Pi^r(c^r, c) \equiv t(c^r) - cx(c^r).$$

represent the firm's profit associated with a regulation contract  $(t, x)$  if it reports a cost  $c^r$  when its actual costs are  $c$ . It then follows that the regulator's optimal contract is a solution to the following maximization problem.

$$P : \max_{x(\cdot), t(\cdot)} \int_{c_l}^{c_h} W(x(c), t(c), c | \lambda, \mu) f(c) dc \quad (2)$$

$$\text{s.t.} \quad \Pi^r(c, c) \geq \Pi^r(c^r, c) \quad \forall c, c^r \in [c_l, c_h] \quad (3)$$

$$\Pi^r(c, c) \geq 0, \quad \forall c \in [c_l, c_h], \quad (4)$$

where (3) represents the incentive compatibility conditions that ensure truthtelling and (4) represents the firm's participation constraints.

Solving the regulator's problem is standard. Incentive compatibility implies that  $x(c)$  is weakly decreasing in  $c$ . As a consequence, any incentive compatible schedule  $x(c)$  is differentiable almost everywhere. Hence, an incentive compatible direct mechanism satisfies the first order condition for truthtelling,

$$\left. \frac{\partial \Pi^r}{\partial c^r} \right|_{c^r=c} = 0 \Rightarrow t'(c) = cx'(c). \quad (5)$$

If we let  $\tilde{\Pi}(c) \equiv \Pi^r(c, c)$  denote the firm's profit when it reports its cost truthfully, then, by the envelope theorem, incentive compatibility implies

$$\tilde{\Pi}'(c) = -x(c).$$

Consequently, the firm's profit is

$$\tilde{\Pi}(c) = \tilde{\Pi}(c_h) - \int_c^{c_h} \tilde{\Pi}'(\tilde{c}) d\tilde{c} = \int_c^{c_h} x(\tilde{c}) d\tilde{c} + \tilde{\Pi}(c_h). \quad (6)$$

Because  $t(c) = \tilde{\Pi}(c) + cx(c)$ , we may, after substitution and an integration by parts, rewrite the regulator's objective function (2) as<sup>8</sup>

$$\tilde{W}(x(c), c|\lambda, \mu) \equiv \int_{c_l}^{c_h} \tilde{w}(x(c), c|\lambda, \mu) f(c) dc, \quad (7)$$

with

$$\tilde{w}(x, c|\lambda, \mu) \equiv v(x) - (1 + \mu)cx - (1 + \mu - \lambda)xh(c). \quad (8)$$

Consequently, the optimal quantity schedule  $x(\cdot)$  solves the problem

$$\begin{aligned} \tilde{P} : \max_{x(\cdot)} \quad & \tilde{W}(x(c), c|\lambda, \mu) \\ \text{s.t.} \quad & x(c) \text{ is monotone decreasing.} \end{aligned} \quad (9)$$

Due to the concavity of  $v(x)$  and the non-decreasing hazard rate  $h(c)$ , the monotonicity constraint is not binding. A solution to  $\tilde{P}$  obtains, therefore, from a point-wise maximization with respect to  $x(c)$ . The first order condition for an optimal schedule  $x^*(c)$  yields<sup>9</sup>

$$v'(x^*(c)) = (1 + \mu)c + (1 + \mu - \lambda)h(c). \quad (10)$$

The optimality condition (10) has the usual interpretation. In the optimum, the regulator equates the consumer's marginal utility to the regulator's virtual cost,  $\bar{c} \equiv (1 + \mu)c + (1 + \mu - \lambda)h(c)$ .

An important benchmark obtains when the cost of public funds is zero ( $\mu = 0$ ) and the regulator values the firm's profits in full ( $\lambda = 1$ ). In this case, there are no distortions. Let

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<sup>8</sup>For the formal exposition

<sup>9</sup>Due to the concavity of  $v$ , first order conditions are sufficient. Sufficient conditions on  $v$  such that (10) has a solution are  $\lim_{x \rightarrow 0} v'(x) = \infty$  and  $\lim_{x \rightarrow \infty} v'(x) = 0$ .

$x^{fb}(c)$  denote the optimal undistorted quantity schedule. That is,  $x^{fb}(c)$  satisfies the first order condition

$$v'(x^{fb}(c)) = c. \quad (11)$$

In the first best, marginal benefits equal the actual marginal costs,  $c$ , rather than the virtual costs,  $\bar{c}$ . If we define

$$\bar{x} \equiv x^{fb}(c_l),$$

we can interpret  $\bar{x}$  as the maximum relevant output level, because the concavity of  $v$  implies that, for any possible combination  $(\lambda, \mu)$ , we have  $x^*(c) \leq x^{fb}(c) \leq \bar{x}$ .

The first order condition (10) will play a crucial role in the subsequent analysis. In order to express explicitly the dependence of the optimal regulatory scheme on the policy variables  $\lambda$  and  $\mu$ , let  $(x^*(\lambda, \mu|c), t^*(\lambda, \mu|c))$  represent the regulation schedule by a regulator who attaches weight  $\lambda$  to profits and cost  $\mu$  to the deadweight loss of taxation. The scheme  $x^*(\lambda, \mu|c)$  is implicitly defined by (10). Using the first order condition for optimal reporting (5) and that, optimally,  $\tilde{\Pi}(c_h) = 0$ , the optimal payment schedule  $t^*(\lambda, \mu|c)$  is

$$t^*(\lambda, \mu|c) = c_h x^*(\lambda, \mu|c_h) - \int_c^{c_h} c \frac{\partial x^*(\lambda, \mu|c)}{\partial c} dc. \quad (12)$$

In the remainder of this section, I derive the properties of the regulatory schedule  $x^*(\lambda, \mu|c)$  that will be helpful to identify the effect of regulatory risk on the different economic agents.

We may first confirm the straightforward intuition that the schedule  $x^*(\lambda, \mu|c)$  is increasing in  $\lambda$  and decreasing in  $\mu$ . Indeed, using the implicit function theorem a differentiation of (10) with respect to  $\lambda$  and  $\mu$  yields

$$v''(x^*(c)) \frac{\partial x^*}{\partial \lambda} = -h(c) \text{ and } v''(x^*(c)) \frac{\partial x^*}{\partial \mu} = c + h(c). \quad (13)$$

From these two equations, it follows  $\partial x^*/\partial \lambda > 0$  and  $\partial x^*/\partial \mu < 0$ .

Next, we address the curvature of  $x^*(\lambda, \mu|c)$ . Differentiating the expressions in (13) with respect to  $\lambda$  and  $\mu$ , it follows, after a rearrangement of terms,

$$\frac{\partial^2 x^*}{\partial \lambda^2} = -\frac{v'''(x^*(c))}{v''(x^*(c))} \left( \frac{\partial x^*}{\partial \lambda} \right)^2 \quad (14)$$

$$\frac{\partial^2 x^*}{\partial \mu^2} = -\frac{v'''(x^*(c))}{v''(x^*(c))} \left( \frac{\partial x^*}{\partial \mu} \right)^2 \quad (15)$$

$$\frac{\partial^2 x^*}{\partial \lambda \partial \mu} = -\frac{v'''(x^*(c))}{v''(x^*(c))} \frac{\partial x^*}{\partial \lambda} \frac{\partial x^*}{\partial \mu}. \quad (16)$$

Hence, the Hessian of  $x^*(\lambda, \mu|c)$  is

$$Dx^*(\lambda, \mu) \equiv \begin{bmatrix} \frac{\partial^2 x^*}{\partial \lambda^2} & \frac{\partial^2 x^*}{\partial \lambda \partial \mu} \\ \frac{\partial^2 x^*}{\partial \lambda \partial \mu} & \frac{\partial^2 x^*}{\partial \mu^2} \end{bmatrix} = -\frac{v'''(x^*(c))}{v''(x^*(c))} \begin{bmatrix} \left( \frac{\partial x^*}{\partial \lambda} \right)^2 & \frac{\partial x^*}{\partial \lambda} \frac{\partial x^*}{\partial \mu} \\ \frac{\partial x^*}{\partial \lambda} \frac{\partial x^*}{\partial \mu} & \left( \frac{\partial x^*}{\partial \mu} \right)^2 \end{bmatrix}.$$

Due to the specific symmetry, the determinant of the Hessian matrix is zero so that  $x^*(c)$  is convex in  $(\lambda, \mu)$  for  $v''' > 0$ . Likewise,  $x^*(c)$  is concave for  $v''' < 0$ . The following proposition collects these insights.

**Proposition 1** *The optimal regulation schedule  $x^*(c)$  is characterized by the first order condition (10). The optimal quantity  $x^*(\lambda, \mu|c)$  is increasing in  $\lambda$  and decreasing in  $\mu$ . It is convex in  $(\lambda, \mu)$  if aggregate demand is convex. It is concave in  $(\lambda, \mu)$  if aggregate demand is concave.*

The intuition why the curvature of the regulatory schedule coincides with the curvature of the demand function is illustrated in Figure 1.<sup>10</sup> Because  $v'(x)$  reflects the consumers' aggregate (inverse) demand function, identity (10) implies that the regulator equates demand to his virtual marginal cost  $\bar{c} = (1 + \mu)c + (1 + \mu - \lambda)h(c)$ . When the policy parameters are random, the virtual costs  $\bar{c}$  are random. Figure 1 illustrates for a two point distribution that, for convex demand, the expected demand,  $x^e = E_{\lambda, \mu}\{x(\bar{c})\}$ , exceeds the demand of the

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<sup>10</sup>Note that the effect of regulatory risk depends exactly on the term  $-v'''(x^*(c))/v''(x^*(c))$ . Interestingly, this ratio is the well-known prudence measure in the literature on precautionary savings (e.g., Leland 1968 and Kimball 1990), where exactly this measure determines whether the consumer increases or decreases its savings in response to additional risk. Here it plays a similar role of determining whether more risk induces the regulator to in- or decrease the output  $x$ .

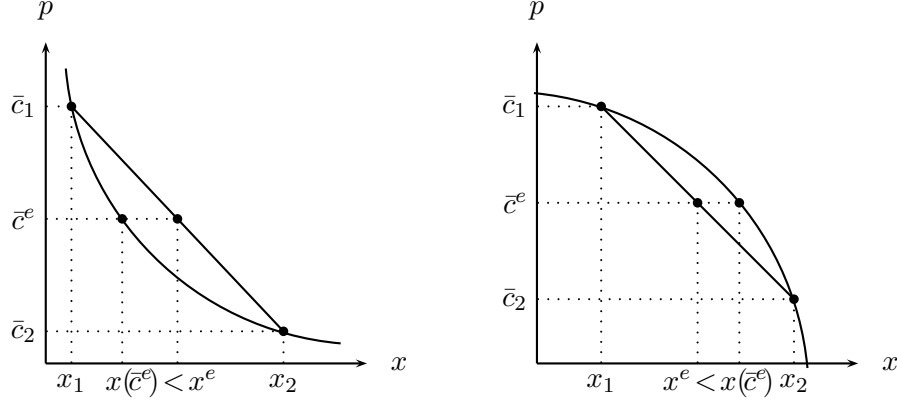


Figure 1: Convex ( $v''' > 0$ ) and concave ( $v''' < 0$ ) demand.

expected virtual costs,  $x(\bar{c}^e) = x^*(E_{\lambda, \mu}\{\bar{c}\})$ . For concave demand, we obtain the opposite result.

## 5 Attitudes Towards Regulatory Risk

In this section, I study the attitudes towards regulatory risk. The central idea is the following. Suppose that the policy parameters  $(\lambda, \mu)$  are distributed according to some non-degenerate distribution  $g(\lambda, \mu)$  with expectations  $(\lambda_e, \mu_e) \equiv E_g(\lambda, \mu)$ . I say that an economic agent *likes* regulatory risk whenever his expected payoffs under any non-degenerate distribution  $g(\lambda, \mu)$  are larger than the payoffs that obtain when the policy parameters are deterministic and correspond to the expectations  $(\lambda_e, \mu_e)$ .<sup>11</sup> Whenever, for any non-degenerate distribution, expected profits are lower than the profits under the deterministic parameters  $(\lambda_e, \mu_e)$ , the economic agent *dislikes* regulatory risk. I study the risk attitude for each of the three types of economic agents: the firm, the regulator, and the consumers.

The attitude towards regulatory risk depends on the curvature of the payoff function with

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<sup>11</sup>Hence, the distribution  $g(\lambda, \mu)$  is a mean preserving spread of the degenerate distribution  $(\lambda_e, \mu_e)$  in the sense of Rothschild and Stiglitz (1970).

respect to the policy variables. Therefore, an economic agent likes regulatory risk exactly when his payoff is convex in the policy parameters  $(\lambda, \mu)$ . In contrast, he dislikes regulatory risk when his payoff is concave.

I first address the firm's risk attitude. From (6) and  $\tilde{\Pi}(c_h) = 0$ , it follows that the profit of a firm with cost  $c$  coincides with its information rent:

$$\tilde{\Pi}(c) = \int_c^{c_h} x^*(\tilde{c}) d\tilde{c}.$$

Hence, the curvature of  $x^*(\lambda, \mu|c)$  determines the curvature of  $\tilde{\Pi}(c)$ . This observation yields the following result.

**Proposition 2** *The firm likes regulatory risk when demand is convex. For concave demand, the firm dislikes regulatory risk.*

The proposition shows that the curvature of the consumers' aggregate demand function fully determines the firm's attitude towards regulatory risk. We may explain this observation on the basis of an *information rent effect* of regulatory risk. Figure 1 shows that, for convex demand, the regulator responds to regulatory risk by raising output in expected terms. Expression (6) shows that, due to the need for incentive compatibility, a raise in output requires larger information rents. Hence, under convex demand regulatory risk has a positive information rent effect, which benefits the firm. In contrast, the regulator responds to regulatory risk with a reduction in output when demand is concave. For concave demand, the information rent effect is, therefore, negative and this hurts the firm.

Next, consider the risk attitude of the regulator. Due to (7), a sufficient condition for the convexity of the regulator's payoff function in  $(\lambda, \mu)$  is that  $\tilde{w}(x, c|\lambda, \mu)$  is convex in  $(\lambda, \mu)$  for all  $c$ .

Because  $x^*(\lambda, \mu|c)$  maximizes  $\tilde{w}(x, c|\lambda, \mu)$  with respect to  $x$ , the envelope theorem yields

$$\frac{d\tilde{w}}{d\lambda} = h(c)x^*(\lambda, \mu|c) \quad \text{and} \quad \frac{d\tilde{w}}{d\mu} = -(c + h(c))x^*(\lambda, \mu|c).$$

Further differentiation yields the Hessian of  $\tilde{w}$  with respect to  $(\lambda, \mu)$ ,

$$D\tilde{w}(\lambda, \mu) \equiv \begin{bmatrix} \frac{\partial^2 \tilde{w}}{\partial \lambda^2} & \frac{\partial^2 \tilde{w}}{\partial \lambda \partial \mu} \\ \frac{\partial^2 \tilde{w}}{\partial \lambda \partial \mu} & \frac{\partial^2 \tilde{w}}{\partial \mu^2} \end{bmatrix} = \begin{bmatrix} h(c) \frac{\partial x^*(c)}{\partial \lambda} & -(c + h(c)) \frac{\partial x^*(c)}{\partial \lambda} \\ h(c) \frac{\partial x^*(c)}{\partial \mu} & -(c + h(c)) \frac{\partial x^*(c)}{\partial \mu} \end{bmatrix}.$$

The Hessian's determinant is zero and, due to  $\partial x^*/\partial \lambda > 0$  and  $\partial x^*/\partial \mu < 0$ , it follows that the Hessian is positive semi-definite. As a result,  $\tilde{w}$  is convex in  $(\lambda, \mu)$  and, therefore, also  $\tilde{W}$  is convex in  $(\lambda, \mu)$ . This yields the following result.

**Proposition 3** *Independent of demand conditions, the regulator likes regulatory risk.*

The intuition behind the proposition is similar to Waugh (1944)'s classical observation that consumers may benefit from fluctuating prices. The connection becomes clear when considering Waugh's argument for the case of a quasi-linear economy in which the consumer has to choose between a consumption good  $x$  and a numeraire  $y$  and has the quasi-linear utility  $u(x, y) = v(x) + y$ . In this case, the Marshallian consumer surplus is an appropriate measure of the consumer's welfare and, from decreasing marginal utility ( $v''(x) < 0$ ), it follows that the Marshallian consumer surplus is convex in the price  $p$  of the consumption good  $x$ . Note the similarity between Waugh's consumer and the regulator in my model: Waugh's consumer maximizes  $v(x) - px$  and the quantity  $x$  satisfies the first order condition  $v'(x) = p$ . Similarly, equation (8) reveals that the regulator maximizes  $v(x) - \bar{c}x$  and the quantity  $x$  satisfies the first order condition (10). The regulator's virtual cost  $\bar{c}$  plays, therefore, a role similar to the price  $p$  in the consumer's problem. As a consequence, the reason why fluctuations in  $(\lambda, \mu)$  benefit the regulator is identical to the reason behind Waugh (1944)'s observation that consumers in standard consumer theory may benefit from fluctuating prices.

Finally, consider the consumer's attitude towards regulatory risk. Here matters are somewhat less straightforward, because, due to decreasing marginal utility ( $v'' < 0$ ), the consumer has already a natural tendency to dislike variations in output. To see how this affects the consumer's overall attitude towards regulatory risk, I first rewrite the consumers' expected utility from the regulator scheme  $x^*$  as

$$\Psi = \int_{c_l}^{c_h} \Psi(t^*(c), x^*(c)) f(c) dc = \int_{c_l}^{c_h} \psi(x^*(c), c | \lambda, \mu) f(c) dc, \quad (17)$$



where

$$\psi(x, c|\lambda, \mu) \equiv v(x) - (1 + \mu)(c + h(c))x. \quad (18)$$

The expression  $\psi$  represents the virtual surplus of  $x$  from the consumers' point of view. They have to incur the transfer  $t$  to the firm which consists of the firm's costs  $c$  and an information rent that equals the hazard rate  $h(c)$ . For  $\mu > 0$ , the virtual marginal cost  $c + h(c)$  is augmented by the distortionary tax  $\mu$ .

Whenever the consumer's virtual surplus  $\psi$  is concave in  $(\lambda, \mu)$ , the consumer's surplus  $\Psi$  is concave in  $(\lambda, \mu)$  and consumers dislike regulatory risk. The curvature of the virtual surplus function  $\psi$  therefore plays a crucial role.

From (18) it follows

$$\frac{\partial^2 \psi}{\partial \lambda^2} = \left[ \frac{v'''(x^*(c))}{v''(x^*(c))} \lambda h(c) + v''(x^*(c)) \right] \left( \frac{\partial x^*(c)}{\partial \lambda} \right)^2; \quad (19)$$

$$\frac{\partial^2 \psi}{\partial \mu^2} = \left[ \frac{v'''(x^*(c))}{v''(x^*(c))} \lambda h(c) - v''(x^*(c)) \right] \left( \frac{\partial x^*(c)}{\partial \mu} \right)^2; \quad (20)$$

and

$$\frac{\partial^2 \psi}{\partial \lambda \partial \mu} = \lambda h(c) \frac{v'''(x^*(c))}{v''(x^*(c))} \frac{\partial x^*(c)}{\partial \lambda} \frac{\partial x^*(c)}{\partial \mu}. \quad (21)$$

From these second order derivatives follows the Hessian of  $\psi$  with determinant

$$- \left( v''(x^*(c)) \frac{\partial x^*(c)}{\partial \lambda} \frac{\partial x^*(c)}{\partial \mu} \right)^2.$$

The determinant is negative for any specification of  $v$  and, therefore, the function  $\psi$  is neither convex nor concave in  $(\lambda, \mu)$ . This shows that, at this level of generality, where one considers uncertainty in both policy variables at the same time, little can be said about the overall risk attitude of consumers.

More insights can be gained, however, when considering uncertainty in each dimension separately. First, consider uncertainty in the policy variable  $\mu$ , which measures the cost

of public funds. Equation (20) reveals that  $\psi$  is convex in  $\mu$  for  $v''' < 0$ . Hence, with concave demand consumers like risk with respect to  $\mu$ . For convex demand, the sign of (20) is indeterminate. To better understand this indeterminacy, observe that the consumers' utility is closely linked to the regulator's utility function:

$$\psi(x, c|\lambda, \mu) = \tilde{w}(x, c|\lambda, \mu) - \lambda x h(c). \quad (22)$$

Hence, the regulator's and the consumers' objective function differ exactly by the term  $\lambda x h(c)$  which reflects the regulator's valuation of the firm's profit. Proposition 3 demonstrates that  $\tilde{w}$  is convex in both  $\lambda$  and  $\mu$ . When demand is concave, the output schedule  $x$  is concave in  $\mu$  and, therefore, the expression  $\lambda x h(c)$  is also concave in  $\mu$ . Subtracting a concave function from a convex function, reinforces the convexity of the latter function. Hence, with concave demand the convexity of  $\tilde{w}$  implies that also  $\psi$  must be convex so that consumers like regulatory risk. In contrast, the expression  $\lambda x h(c)$  counteracts the convexity of  $\tilde{w}$  when demand is convex. It then depends on the relative strength of the two terms whether  $\psi$  is convex or concave. This explains why the curvature of  $\psi$  is indeterminate with convex demand.

Second, consider the consumer's attitude towards uncertainty in the policy variable  $\lambda$ , which measures the weight which the regulator attaches to the firm's profits. Equation (19) reveals that  $\psi$  is concave in  $\lambda$  for  $v''' > 0$ . For  $v''' < 0$ , the function  $\psi$  may either be convex or concave in  $\lambda$ . We can explain this result by reconsidering the curvature of the expression  $\lambda x h(c)$  in (22) but now with respect to  $\lambda$ . Taking account that the expression depends on  $\lambda$  directly and indirectly via  $x$ , the second order derivative with respect to  $\lambda$  reveals that the curvature depends on the sign of

$$\frac{2v''(x) - v'''(x)}{v''(x)} \frac{\partial x}{\partial \lambda} h(c).$$

For convex demand ( $v''' > 0$ ), the sign is negative and, therefore, the term,  $\lambda x h(c)$ , is concave. Subtracting it from  $\tilde{w}$  then reinforces its convexity and, as a consequence,  $\psi$  is convex itself. We, therefore, obtain that consumers like risk in  $\lambda$ . In contrast, the expression may be convex when demand is concave so that it counteracts the convexity of  $\tilde{w}$ . In this case, the curvature of  $\psi$  with respect to  $\lambda$  is ambiguous.

Table II

Attitudes towards regulatory risk						
	Risk in $\lambda$			Risk in $\mu$		
	Firm	Regulator	Consumer	Firm	Regulator	Consumer
convex demand	+	+	-	+	+	-/+
concave demand	-	+	-/+	-	+	+

The following proposition collect these observations.

**Proposition 4** *With convex demand, consumers dislike regulatory risk concerning  $\lambda$ . With concave demand, consumers like regulatory risk concerning  $\mu$ .*

The proposition shows that consumers tend to dislike regulatory risk when demand is convex and tend to like it for concave demand. Yet, in general, the curvature of the demand function is insufficient to determine the consumer's attitude towards regulatory risk. This indeterminacy obtains because, due to the consumer's decreasing marginal utility, the consumer has a natural tendency to dislike fluctuations in the output  $x$ . Hence, the consumer's attitude towards regulatory risk depends on two factors: the information rent effect and the way regulatory risk affect fluctuations in the overall output schedule  $x(c)$ . The curvature of the demand function determines only the first effect, whereas the second effect depends also on  $v''(x)$  which represents the slope of the demand function.

Table II summarizes the main findings of Proposition 2, 3, and 4. It illustrates the risk attitudes of the three economic parties in the different policy dimensions. Because of a contradictory evaluation of the information rent effect of regulatory risk, the risk attitudes of the firm and consumers tend to be misaligned. Firms like risk when demand is convex, but in this case consumer dislike risk concerning  $\lambda$ . Similarly, with concave demand, firms dislike risk whereas consumer like risk with respect to  $\mu$ . Yet, when the information rent effect is too small to outweigh the consumers' natural tendency to dislike variations in output due to decreasing marginal utility, the firm's and consumers' risk preferences coincide. In

particular, when demand is concave, both the firm and consumers may dislike regulatory risk. Because the regulators always likes regulatory risk, we obtain, therefore, the following, surprising result.

**Proposition 5** *A benevolent regulator's preferences concerning regulatory risk with respect to  $\lambda$  contradict the preferences of both the firm and consumers whenever for all  $x \in (0, \bar{x}]$  and  $c \in [c_l, c_h]$ ,*

$$v'''(x) < 0 \text{ and } |v'''(x)|h(c) < [v''(x)]^2. \quad (23)$$

At first sight, this is a paradoxical result. How can a benevolent regulator, who has only the interest of the firm and consumers in mind, have preferences that go against the interests of both the firm and consumers? Indeed, the objective function of the regulator is a weighted sum of the firm's and the consumers' payoffs. Hence, based on the mathematical fact, that a weighted sum of two concave functions is itself concave, one is tempted to argue that if both the firm and the consumers dislike regulatory risk, then also the regulator must dislike it. Yet, this reasoning neglects that, under regulatory risk in  $\lambda$ , also the weight itself involves risk. This yields an additional effect of risk on the regulator that is not present for the firm or consumers. The proposition shows that condition (23) is sufficient for this additional effect to dominate.<sup>12</sup> Intuitively, the condition says that demand should be concave but not too much. More specifically, it requires that  $|v'''|$  is relatively small in comparison to  $|v''|$ . We can explain this insight by referring to the information rent effect of regulatory risk and the consumers' natural tendency to dislike risk due to decreasing marginal utility: The term  $|v'''|$  measures the strength of the information rent effect, whereas  $|v''|$  measures the strength of the consumers' decreasing marginal utility. Hence, a relatively small  $|v'''|$  in comparison to

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<sup>12</sup>A simple, concrete example where the effect dominates is  $v(x) = -x^3 + 2x^2 + 2x$ , marginal cost  $c$  uniformly distributed over  $[1, 2]$ , and uncertainty about policy variable  $\lambda$  only. Because the effect dominates already for rather straightforward specifications of the model, Proposition 5 does not seem to reflect only pathological cases.

$|v''|$  implies that the information rent effect is too small to counteract the natural tendency of the consumer to dislike fluctuations in output due to the consumers' decreasing marginal utility. Hence, despite the information rent effect, consumers still dislike regulatory risk. In addition, a negative  $v'''$  means that the firm also dislikes regulatory risk. Because the regulator always likes regulatory risk, the preferences of the firm and consumers contradict those of the regulator.

Table II reveals that, for risk in  $\mu$  only, it is not possible that the risk attitudes of both the firm and the consumers contradict the regulator's attitude towards regulatory risk. This is exactly because the regulator's objective function is a weighted sum of the firm's and the consumer's preferences. If the firm's and consumers' risk preferences concerning  $\mu$  are aligned then the regulator's preferences also coincide with these preferences. Hence, a misalignment of the regulator's preferences can obtain only when the riskiness in  $\lambda$  is strong enough relative to the riskiness in  $\mu$ .

The opposing risk preferences have the important economic implication that it may induce a benevolent regulator to act against the interest of both consumers and the regulated firm. In particular, a regulator will not take (costly) measures to lower the degree of regulatory risk. Indeed, to induce the regulator to take such measures and act in the interest of the firm and consumers, other explicit incentives are needed. An example would be an explicit directive for the regulator to reduce regulatory risk. From Proposition 5, it follows that it does not suffice to endow the regulator with a payoff function that is a weighted sum of the parties he is to represent.

## 6 Conclusion and Discussion

This paper studies the effect of regulatory risk in an optimal regulatory framework on the three different economic agents: firms, consumers, and regulators. The analysis yields the following insights and policy implications.

First, when demand is convex, regulatory risk leads to larger expected profits. Taking

expected future profits as a measure of the incentive to invest, the larger expected profits suggest a stronger investment incentive for a regulated firm. This result contradicts the common view that regulatory risk is detrimental to investment and economic growth and should therefore be avoided. Conclusions are, however, reversed for concave demand. With regard to investment incentives and economic growth, the analysis reveals that the curvature of demand plays a crucial role for policy implications.

Second, the consumers' risk attitudes tend to contradict the firm's attitude towards regulatory risk. In particular, when demand is convex, consumers dislike risk about the relative weight at which the regulator values profits. When demand is concave, consumers like risk about the cost of public funds. In some circumstances, however, the attitudes of the firm and consumers coincide. In particular, for concave demand regulatory risk may hurt both the firm and consumers. In this case, the policy implication is to minimize regulatory risk.

Third, irrespective of the firm's and the consumers' risk attitudes, the regulator always likes regulatory risk. Hence, with regard to regulatory risk, one cannot expect the regulator to act in society's interest even if his objective function is a weighted sum of the consumers' and the firm's surplus. This insight yields the policy implication that regulators may require explicit incentives to minimize regulatory risk. In particular, when demand is concave and regulatory risk hurts both the firm and consumers, the regulator should be given an explicit directive to reduce regulatory risk. A public communication of the UK Office of Rail Regulation in 2000 provides some anecdotal evidence of a regulator's relaxed attitude towards regulatory risk.<sup>13</sup> The letter, written by the Chief Economist after public concerns about regulatory risk, acknowledges its potential threat to effective regulation but downplays its actual significance.

Results show that the curvature of the demand function is a major determinant of the economic effects of regulatory risk. Regrettably, the economic literature provides little infor-

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<sup>13</sup>See "What Regulatory Risk" by Paul Plummer, ORR Chief Economist, January 2000, accessible at <http://www.rail-reg.gov.uk/upload/pdf/utiljrnl.pdf> (last retrieved 11-15-2008).

mation about the curvature of demand functions. First, consumer theory does not provide any guidance about curvature. Indeed, in economic theory some applications use concave demand, while others assume it to be convex.<sup>14</sup> Hence, the question whether demand is convex or concave is a purely empirically one and depends on the specific market under consideration. Yet, even though the empirical literature on demand functions is vast, there are few empirical results about the curvature of demand. One reason is that most empirical studies are based on parametric demand estimations. These estimation methods presume already a specific functional form that implicitly determines the shape of the demand function. A popular parametric specification is, for instance, the log-linear one, which implicitly presumes a convex demand function. Given the lack of concrete empirical evidence, more empirical work on the shape of demand functions would be helpful both for applying the paper's theoretical results and for testing its predictions empirically.

The formal exposition focused on a constant return to scale technology. This assumption is not crucial. The results are, for example, readily extendable to any cost function under the condition that the firm's type enters multiplicatively. To make this more precise, define a cost function  $c(x, \theta)$  as *multiplicatively separable* if there exist two functions  $g(\cdot)$  and  $k(\cdot)$  with  $g' \geq 0$  and  $k' \geq 0$  such that  $c(x, \theta) = g(\theta)k(x)$ . One may reformulate any multiplicatively separable cost function in terms of  $\hat{\theta}$  and  $\hat{x}$  with  $\hat{\theta} = g(\theta)$  and  $\hat{x} = k(x)$ . Indeed, because  $\hat{c}(\hat{x}, \hat{\theta}) = \hat{\theta}\hat{x} = g(\theta)k(x) = c(x, \theta)$ , the transformation yields a linear cost function in terms of the transformed variables  $\hat{x}$  and  $\hat{\theta}$ . Defining  $\hat{v}(\hat{x}) \equiv v(k^{-1}(\hat{x}))$ , it follows that the third derivative of  $\hat{v}(\cdot)$  now determines risk attitudes. Because  $\hat{v}(\cdot)$  is a composite function of both  $v(\cdot)$  and  $k(\cdot)$ , attitudes depend both on consumer surplus and the firm's cost function.

The present paper points to numerous extensions that would further enhance our understanding of regulatory risk. For instance, the current framework considered regulatory

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<sup>14</sup>E.g., monopoly and oligopoly theory often assume concave demand functions in order to ensure that the firm's maximization problem is well defined and solutions are well behaved. In other studies, economic theory often assumes that, at least locally, elasticities are constant. Constant elasticities imply convex demand. Convex demand also obtains, if aggregate demand is derived from linear individual demand curves with different intercepts; a typical exercise in standard text books.

risk as unalterable and exogenous. An obvious critique is that this view of regulatory risk is rather limited. Indeed, already the introduction motivated regulatory risk with changes in political environments and tax systems. Clearly, such changes are not simply random but depend on the behavior of economic agents. Hence, an obvious and important question is how economic agent can and should manage regulatory risk. However, before one can address such questions, a necessary first step is to understand the effect of regulatory risk when it is exogenous. The contribution of the present paper is to provide this step.

Moreover, the paper models regulatory risk as uncertainty about the regulator's objective function and, in particular, about two policy variables: the relative weight attached to profits and the cost of public funds. Per definition, regulatory risk is any uncertainty about future regulation and this uncertainty may have other causes than the ones examined here. For instance, uncertainties about the regulator's commitment to long term contracts (Laffont and Tirole 1988) or his susceptibility to renegotiation (Laffont and Tirole 1990) may lead to regulatory risk. In less developed countries, uncertainties about the rule of law and the threat of expropriation may further exacerbate regulatory risk. Given the many different origins of regulatory risk, a fruitful direction for future research is towards a comprehensive classification of different causes and their consequences. From this perspective, the contribution of this paper lies in providing a framework to analyze a specific class of regulatory risk: uncertainty about the regulator's policy variables.



## Appendix

**Proof of Proposition 1:** Follows directly from the body text.

Q.E.D.

**Proof of Proposition 2:** It follows  $\partial^2 \tilde{\Pi}(c)/\partial \lambda^2 = \int_c^{c_h} \partial^2 x^*(\tilde{c})/\partial \lambda^2 d\tilde{c}$  and  $\partial^2 \tilde{\Pi}(c)/\partial \lambda \partial \mu = \int_c^{c_h} \partial^2 x^*(\tilde{c})/\partial \lambda \partial \mu d\tilde{c}$  and  $\partial^2 \tilde{\Pi}(c)/\partial \mu^2 = \int_c^{c_h} \partial^2 x^*(\tilde{c})/\partial \mu^2 d\tilde{c}$ . Therefore,  $\tilde{\Pi}(c)$  is convex in  $(\lambda, \mu)$  if  $x^*(c)$  is convex in  $(\lambda, \mu)$  for all  $c \in (c, c_h)$  and  $\tilde{\Pi}(c)$  is concave in  $(\lambda, \mu)$  if  $x^*(c)$  is concave in  $(\lambda, \mu)$  for all  $c \in (c, c_h)$ . From Proposition 1 it then follows that  $\tilde{\Pi}(c)$  is convex in  $(\lambda, \mu)$  whenever aggregate demand is convex ( $v''' > 0$ ) and  $\tilde{\Pi}(c)$  is concave in  $(\lambda, \mu)$  whenever aggregate demand is concave ( $v''' < 0$ ).

Q.E.D.

**Proof of Proposition 3:** It follows  $\partial^2 \tilde{W}/\partial \lambda^2 = \int_{c_l}^{c_h} \partial^2 \tilde{w}(\tilde{c})/\partial \lambda^2 d\tilde{c}$  and  $\partial^2 \tilde{W}/\partial \lambda \partial \mu = \int_{c_l}^{c_h} \partial^2 \tilde{w}(\tilde{c})/\partial \lambda \partial \mu d\tilde{c}$  and  $\partial^2 \tilde{W}/\partial \mu^2 = \int_{c_l}^{c_h} \partial^2 \tilde{w}(\tilde{c})/\partial \mu^2 d\tilde{c}$ . Therefore,  $\tilde{W}$  is convex in  $(\lambda, \mu)$  if  $\tilde{w}$  is convex in  $(\lambda, \mu)$  for all  $c \in (c_l, c_h)$ , which is the case because the Hessian  $D\tilde{w}(\lambda, \mu)$  is positive semi-definite.

Q.E.D.

**Proof of Proposition 4:** It follows  $\partial^2 \Psi/\partial \lambda^2 = \int_{c_l}^{c_h} \partial^2 \psi(x^*(\tilde{c}), \tilde{c})/\partial \lambda^2 d\tilde{c}$  and  $\partial^2 \Psi/\partial \mu^2 = \int_{c_l}^{c_h} \partial^2 \psi(x^*(\tilde{c}), \tilde{c})/\partial \mu^2 d\tilde{c}$ . Therefore,  $\Psi$  is convex in  $\lambda$  if  $\psi(x^*(\tilde{c}), \tilde{c})$  is convex in  $\lambda$  for all  $c \in (c_l, c_h)$  and  $\Psi$  is concave in  $\mu$  if  $\psi(x^*(\tilde{c}), \tilde{c})$  is concave in  $\mu$  for all  $c \in (c_l, c_h)$ . Expression (19) shows that  $v''' > 0$  is a sufficient condition for  $\psi(x^*(\tilde{c}), \tilde{c})$  to be convex in  $\lambda$ . Expression (20) shows that  $v''' < 0$  is a sufficient condition for  $\psi(x^*(\tilde{c}), \tilde{c})$  to be concave in  $\mu$ .

Q.E.D.

**Proof of Proposition 5:** According to Proposition 3 the regulator always likes regulatory risk and, therefore, in particular with respect to  $\lambda$  and under condition (23). According to Proposition 2, the condition  $v'''(x) < 0$  for all  $x \leq \bar{x}$  and  $c \in [c_l, c_h]$  implies that  $\Pi(c)$  is concave in  $\lambda$  for all  $c \in [c_l, c_h]$ . Hence, regardless of the firm's cost-type, the firm dislikes regulatory risk with respect to  $\lambda$ . Finally, condition (23) implies, due to  $\lambda \in [0, 1]$ , that  $|v'''(x)|\lambda h(c) \leq |v'''(x)|h(c) < [v''(x)]^2$ . According to (19), it therefore follows with  $v'''(x) < 0$  that  $\partial^2 \psi/\partial \lambda^2 < 0$  for all  $x \in (0, \bar{x})$  and  $c \in [c_l, c_h]$  so that, by (17) and  $x^*(\lambda, \mu) \leq \bar{x}$ , we have  $\partial^2 \Psi/\partial \lambda^2 = \int_{c_l}^{c_h} \partial^2 \psi/\partial \lambda^2 dc < 0$ . This implies that the consumers dislike regulatory risk with respect to  $\lambda$ . Hence, the risk preferences of the regulator contradict those of the firm and consumers.

Q.E.D.

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## Formal Derivations (Not for Publication)

Derivation of (7):

$$W = \int_{c_l}^{c_h} W(x(c), t(c), c | \lambda, \mu) dF(c) \quad (24)$$

$$= \int_{c_l}^{c_h} v(x(c)) - \lambda cx(c) - (1 + \mu - \lambda)t(c) dF(c) \quad (25)$$

$$= \int_{c_l}^{c_h} \{v(x(c)) - \lambda cx(c) - (1 + \mu - \lambda)(\tilde{\Pi}(c) + cx(c))\} f(c) dc \quad (26)$$

$$= \int_{c_l}^{c_h} \{v(x(c)) - (1 + \mu)cx(c) - (1 + \mu - \lambda)\tilde{\Pi}(c)\} f(c) dc \quad (27)$$

$$= \int_{c_l}^{c_h} \{v(x(c)) - (1 + \mu)cx(c)\} f(c) dc \quad (28)$$

$$- (1 + \mu - \lambda) \int_{c_l}^{c_h} \tilde{\Pi}(c) f(c) dc \quad (29)$$

$$= \int_{c_l}^{c_h} \{v(x(c)) - (1 + \mu)cx(c)\} f(c) dc \quad (30)$$

$$- (1 + \mu - \lambda) \left[ [F(c)\Pi(c)]_{c=c_l}^{c=c_h} - \int_{c_l}^{c_h} \tilde{\Pi}'(c) F(c) dc \right] \quad (31)$$

$$= \int_{c_l}^{c_h} \{v(x(c)) - (1 + \mu)cx(c)\} f(c) dc \quad (32)$$

$$+ (1 + \mu - \lambda) \left[ \int_{c_l}^{c_h} x(c) F(c) dc \right] \quad (33)$$

$$= \int_{c_l}^{c_h} \{v(x(c)) - (1 + \mu)cx(c) - (1 + \mu - \lambda)x(c)h(c)\} f(c) dc. \quad (34)$$

Derivation of (12):

$$t(c) = \tilde{\Pi}(c) + cx(c) \quad (35)$$

$$= - \int_{c_l}^{c_h} \tilde{\Pi}'(c) + x(c) + cx'(c) dc + t(c_h) \quad (36)$$

$$= - \int_{c_l}^{c_h} cx'(c) dc + t(c_h) \quad (37)$$

$$= - \int_c^{c_h} c \frac{\partial x^*(\lambda, \mu | c)}{\partial c} dc - c_h x^*(\lambda, \mu | c_h). \quad (38)$$

Derivation of (14): Differentiation of the left identity in (13) with respect to  $\lambda$  yields

$$v'''(x^*(c)) \left( \frac{\partial x^*}{\partial \lambda} \right)^2 + v''(x^*(c)) \frac{\partial^2 x^*}{\partial \lambda^2} = 0 \quad (39)$$

$$\Leftrightarrow \frac{\partial^2 x^*}{\partial \lambda^2} = - \frac{v'''(x^*(c))}{v''(x^*(c))} \left( \frac{\partial x^*}{\partial \lambda} \right)^2 \quad (40)$$

**Derivation of (15):** Differentiation of the right identity in (13) with respect to  $\mu$  yields

$$v'''(x^*(c)) \left( \frac{\partial x^*}{\partial \mu} \right)^2 + v''(x^*(c)) \frac{\partial^2 x^*}{\partial \mu^2} = 0 \quad (41)$$

$$\Leftrightarrow \frac{\partial^2 x^*}{\partial \mu^2} = -\frac{v'''(x^*(c))}{v''(x^*(c))} \left( \frac{\partial x^*}{\partial \mu} \right)^2 \quad (42)$$

**Derivation of (16):** Differentiation of the left identity in (13) with respect to  $\mu$  yields

$$v'''(x^*(c)) \frac{\partial x^*}{\partial \mu} \frac{\partial x^*}{\partial \mu} + v''(x^*(c)) \frac{\partial^2 x^*}{\partial \lambda \partial \mu} = 0 \quad (43)$$

$$\Leftrightarrow \frac{\partial^2 x^*}{\partial \lambda \partial \mu} = -\frac{v'''(x^*(c))}{v''(x^*(c))} \frac{\partial x^*}{\partial \mu} \frac{\partial x^*}{\partial \mu}. \quad (44)$$

**Derivation of (17):**

$$\Psi = \int_{c_l}^{c_h} \Psi(t^*(c), x^*(c)) f(c) dc \quad (45)$$

$$= \int_{c_l}^{c_h} (v(x^*(c)) - (1 + \mu)t^*(c)) f(c) dc \quad (46)$$

$$= \int_{c_l}^{c_h} (v(x^*(c)) - (1 + \mu)[\tilde{\Pi}(c) + cx^*(c)]) f(c) dc \quad (47)$$

$$= \int_{c_l}^{c_h} (v(x^*(c)) - (1 + \mu)[x^*(c)h(c) + cx^*(c)]) f(c) dc \quad (48)$$

$$= \int_{c_l}^{c_h} [v(x^*(c)) - (1 + \mu)(c + h(c))x^*(c)] f(c) dc \quad (49)$$

$$= \int_{c_l}^{c_h} \psi(x^*(c), c|\lambda, \mu) f(c) dc, \quad (50)$$

where the fourth equality uses an integration by parts as in (30).

**Derivation of (19):** Differentiating (18) with respect to  $\lambda$  yields

$$\frac{\partial \psi}{\partial \lambda} = [v'(x^*(c)) - (1 + \mu)(c + h(c))] \frac{\partial x^*(c)}{\partial \lambda}. \quad (51)$$

A further differentiation of (51) with respect to  $\lambda$  and using (10) and (14) yields

$$\frac{\partial^2 \psi}{\partial \lambda^2} = v''(x^*) \left( \frac{\partial x^*}{\partial \lambda} \right)^2 + (v'(x^*) - (1 + \mu)(c + h(c))) \frac{\partial^2 x^*}{\partial \lambda^2} \quad (52)$$

$$= v''(x^*) \left( \frac{\partial x^*}{\partial \lambda} \right)^2 - \lambda h(c) \frac{\partial^2 x^*}{\partial \lambda^2} \quad (53)$$

$$= \left[ \frac{v'''(x^*)}{v''(x^*)} \lambda h(c) + v''(x^*) \right] \left( \frac{\partial x^*}{\partial \lambda} \right)^2. \quad (54)$$

**Derivation of (20):** Differentiating (18) with respect to  $\mu$  yields

$$\frac{\partial \psi}{\partial \mu} = [v'(x^*) - (1 + \mu)(c + h(c))] \frac{\partial x^*}{\partial \mu} - (c + h(c))x^*. \quad (55)$$

A further differentiation of (55) with respect to  $\mu$  and using the right hand equation in (13) and (10) and, finally, (15) yields

$$\frac{\partial^2 \psi}{\partial \mu^2} = v''(x^*) \left( \frac{\partial x^*}{\partial \mu} \right)^2 - 2(c + h(c)) \frac{\partial x^*}{\partial \mu} \quad (56)$$

$$+ (v'(x^*) - (1 + \mu)(c + h(c))) \frac{\partial^2 x^*}{\partial \mu^2} \quad (57)$$

$$= v''(x^*) \left( \frac{\partial x^*}{\partial \mu} \right)^2 - 2v''(x^*) \left( \frac{\partial x^*}{\partial \mu} \right)^2 - \lambda h(c) \frac{\partial^2 x^*}{\partial \mu^2} \quad (58)$$

$$= -v''(x^*) \left( \frac{\partial x^*}{\partial \mu} \right)^2 - \lambda h(c) \frac{\partial^2 x^*}{\partial \mu^2} \quad (59)$$

$$= \left[ \frac{v'''(x^*)}{v''(x^*)} \lambda h(c) - v''(x^*) \right] \left( \frac{\partial x^*}{\partial \mu} \right)^2. \quad (60)$$

**Derivation of (21):** Further differentiation of (51) with respect to  $\mu$  and using (10) and the right hand equation in (13) and, finally, (16) yields

$$\frac{\partial^2 \psi}{\partial \lambda \partial \mu} = v''(x^*) \frac{\partial x^*}{\partial \lambda} \frac{\partial x^*}{\partial \mu} + [v'(x^*) - (1 + \mu)(c + h(c))] \frac{\partial^2 x^*}{\partial \lambda \partial \mu} \quad (61)$$

$$- (c + h(c)) \frac{\partial x^*}{\partial \lambda} \quad (62)$$

$$= v''(x^*) \frac{\partial x^*}{\partial \lambda} \frac{\partial x^*}{\partial \mu} - \lambda h(c) \frac{\partial^2 x^*}{\partial \lambda \partial \mu} - v''(x^*) \frac{\partial x^*}{\partial \mu} \frac{\partial x^*}{\partial \lambda} \quad (63)$$

$$= \lambda h(c) \frac{v'''(x^*)}{v''(x^*)} \frac{\partial x^*}{\partial \lambda} \frac{\partial x^*}{\partial \mu} \quad (64)$$

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